Chapter 12, Conceptual Questions

12.1 As suggested by the figure, we will assume that the larger sphere is more massive. Then the center of gravity would be at point a because if we suspend the dumbbell from point a then the counterclockwise torque due to the large sphere (large weight times small lever arm) will be equal to the clockwise torque due to the small sphere (small weight times large lever arm).
Look at the figure and mentally balance the dumbbell on your finger; your finger would have to be at point a.
The sun-earth system is similar to this except that the sun's mass is so much greater than the earth's that the center of mass (called the barycenter for astronomical objects orbiting each other) is only 450 km from the center of the sun.

12.2. To double the rotational energy without changing ω requires doubling the moment of inertia. The moment of inertia is proportional to R^2 so R must increase by $\sqrt{2}$.

12.3. The rotational kinetic energy is $K_{\text{rot}} = \frac{1}{2}I\omega^2$. For a disk, $I = \frac{1}{2}MR^2$. Since the mass is the same for all three disks, the quantity $R^2\omega^2$ determines the ranking. Thus $K_{\text{a}} = K_{\text{b}} > K_{\text{c}}$.

12.4. No. The moment of inertia does not have any dependence on a quantity that indicates an object is rotating, such as ω or α, so an object does not have to be rotating to have a moment of inertia.

12.5. Mass that is further away from the axis of rotation contributes more to the moment of inertia $I = \int r^2 dm$. Here, r is the distance from the axis of rotation to the mass element dm. Note r^2 is always positive. For a rod, there is more mass further away from an axis through the rod's end than one through its middle.

12.6. Because sphere 2 has twice the radius, its mass is greater by a factor of $2^3 = 8$, since $m = \frac{4}{3}\pi r^3 \rho_{\text{steel}}$. The added mass is also distributed further from the center and, so, $I \propto mr^2$ leads to $I_2 \propto (8m_1)(2r_1)^2 = 32I_1$.

12.7. It will be easier to rotate the solid sphere because the hollow sphere's mass is generally distributed further from its center. If you roll both simultaneously down an incline, the solid sphere will win.

12.8. $\tau_{\text{e}} > \tau_{\text{a}} = \tau_{\text{b}} > \tau_{\text{c}} = \tau_{\text{d}} > \tau_{\text{f}}$ The torque $\tau = rF\left|\sin\theta\right|$. We must calculate each torque:

$$\tau_{\text{a}} = \left(\frac{L}{2}\right)F \qquad \tau_{\text{d}} = \frac{L}{2}F\sin 45^\circ = \frac{\sqrt{2}}{4}LF$$

$$\tau_{\text{b}} = \left(\frac{L}{4}\right)F \qquad \tau_{\text{e}} = L(2F)$$

$$\tau_{\text{c}} = \left(\frac{L}{2}\right)F\sin 45^\circ = \frac{\sqrt{2}}{4}LF \qquad \tau_{\text{f}} = LF\sin 0^\circ = 0$$

12.12. The block attached to the solid cylinder hits first. The solid cylinder has a smaller moment of inertia since more of its mass is closer to the rotation axis, so has less resistance to a change in its rotational motion. The torque applied by the string attached to the block makes the solid cylinder change its rotation and unwind the string faster.

12.13. The moment of inertia for the tuck position is smaller than that of the pike position. Since the angular momentum of the diver is conserved, any initial angular velocity is increased more when the diver moves to the tuck position relative to the pike position.

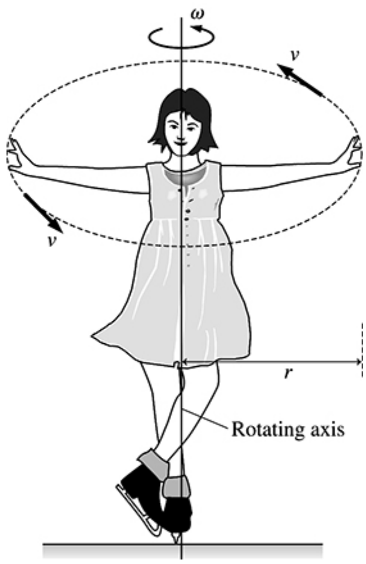
12.14. The angular momentum L of disk b is larger than the angular momentum of disk a. Calculate L for each:

$$L_1 = I_1\omega_1 = \frac{1}{2}mr_1^2\omega_1$$

$$L_2 = I_2\omega_2 = \frac{1}{2}m\left(2r_1^2\right)^2\left(\frac{1}{2}\omega_1\right) = 2\left(\frac{1}{2}mr_1^2\omega_1\right) = 2L_1$$

Chapter 12, Exercises and Problems

12.1. Model: A spinning skater, whose arms are outstretched, is a rigid rotating body.
Visualize:



Solve: The speed $v = r\omega$, where $r = 140\text{ cm}/2 = 0.70\text{ m}$. Also, $180\text{ rpm} = (180)2\pi/60\text{ rad/s} = 6\pi\text{ rad/s}$. Thus, $v = (0.70\text{ m})(6\pi\text{ rad/s}) = 13.2\text{ m/s}$.
Assess: A speed of $13.2\text{ m/s} \approx 26\text{ mph}$ for the hands is a little high, but reasonable.

12.2. Model: Assume constant angular acceleration.

Solve: (a) The final angular velocity is $\omega_\text{f} = (2000\text{ rpm})\left(\dfrac{2\pi\text{ rad}}{\text{rev}}\right)\left(\dfrac{\text{min}}{60\text{ s}}\right) = 209.4\text{ rad/s}$. The definition of angular acceleration gives us

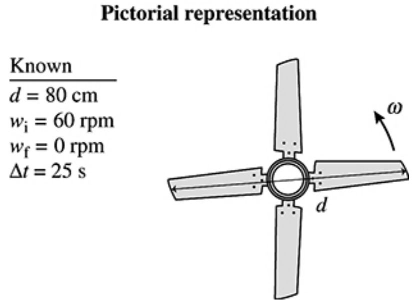
$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_\text{f} - \omega_\text{i}}{\Delta t} = \frac{209.4\text{ rad/s} - 0\text{ rad/s}}{0.50\text{ s}} = 419\text{ rad/s}$$

The angular acceleration of the drill is 4.2×10^2 rad/s.
(b)

$$\theta_\text{f} = \theta_\text{i} + \omega_i\Delta t + \frac{1}{2}\alpha(\Delta t)^2 = 0\text{ rad} + 0\text{ rad} + \frac{1}{2}(419\text{ rad/s})(0.50\text{ s})^2 = 52.4\text{ rad}$$

The drill makes $(52.4\text{ rad})\left(\dfrac{\text{rev}}{2\pi\text{ rad}}\right) = 8.3$ revolutions.

12.4. Model: Assume constant angular acceleration.
Visualize:



Solve: The initial angular velocity is $\omega_\text{i} = (60\text{ rpm})\left(\dfrac{2\pi\text{ rad}}{\text{rev}}\right)\left(\dfrac{\text{min}}{60\text{ s}}\right) = 2\pi\text{ rad/s}$.

The angular acceleration is

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{0 \text{ rad/s} - 2\pi \text{ rad/s}}{25 \text{ s}} = -0.251 \text{ rad/s}^2$$

The angular velocity of the fan blade after 10 s is

$$\omega_f = \omega_i + \alpha(t - t_0) = 2\pi \text{ rad/s} + \left(-0.251 \text{ rad/s}^2\right)(10 \text{ s} - 0 \text{ s}) = 3.77 \text{ rad/s}$$

The tangential speed of the tip of the fan blade is

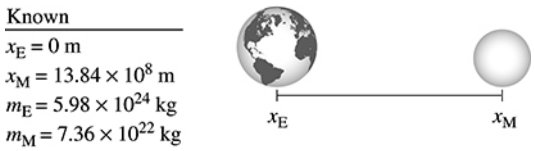
$$v_t = r\omega = (0.40 \text{ m})(3.77 \text{ rad/s}) = 1.51 \text{ m/s}$$

(b)

$$\theta_f = \theta_i + \omega_i \Delta t + \frac{1}{2}\alpha(\Delta t)^2 = 0 \text{ rad} + (2\pi \text{ rad/s})(25 \text{ s}) + \frac{1}{2}\left(-0.251 \text{ rad/s}^2\right)(25 \text{ s})^2 = 78.6 \text{ rad}$$

The fan turns $78.6 \text{ radians} = 12.5 \text{ revolutions}$ while coming to a stop.

12.5. Model: The earth and moon are particles.
Visualize:



Choosing $x_E = 0 \text{ m}$ sets the coordinate origin at the center of the earth so that the center of mass location is the distance from the center of the earth.
Solve:

$$x_{cm} = \frac{m_E x_E + m_M x_M}{m_E + m_M} = \frac{\left(5.98 \times 10^{24} \text{ kg}\right)(0 \text{ m}) + \left(7.36 \times 10^{22} \text{ kg}\right)\left(3.84 \times 10^8 \text{ m}\right)}{5.98 \times 10^{24} \text{ kg} + 7.36 \times 10^{22} \text{ kg}}$$
$$= 4.67 \times 10^6 \text{ m}$$

Assess: The center of mass of the earth-moon system is called the barycenter, and is located beneath the surface of the earth. Even though $x_E = 0 \text{ m}$ the earth influences the center of mass location because m_E is in the denominator of the expression for x_{cm}.

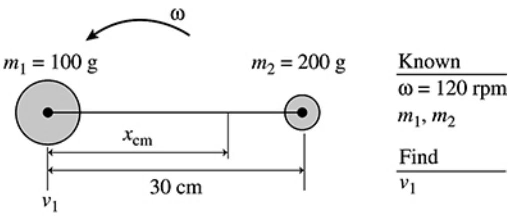
12.6. Visualize: Please refer to Figure EX12.6. The coordinates of the three masses m_A, m_B, and m_C are (0 cm, 0 cm), (0 cm, 10 cm), and (10 cm, 0 cm), respectively.
Solve: The coordinates of the center of mass are

$$x_{cm} = \frac{m_A x_A + m_B x_B + m_C x_C}{m_A + m_B + m_C} = \frac{(100 \text{ g})(0 \text{ cm}) + (200 \text{ g})(0 \text{ cm}) + (300 \text{ g})(10 \text{ cm})}{(100 \text{ g} + 200 \text{ g} + 300 \text{ g})} = 5.0 \text{ cm}$$
$$y_{cm} = \frac{m_A y_A + m_B y_B + m_C y_C}{m_A + m_B + m_C} = \frac{(100 \text{ g})(0 \text{ cm}) + (200 \text{ g})(10 \text{ cm}) + (300 \text{ g})(0 \text{ cm})}{(100 \text{ g} + 200 \text{ g} + 300 \text{ g})} = 3.3 \text{ cm}$$

12.7. Visualize: Please refer to Figure EX12.7. The coordinates of the three masses m_A, m_B, and m_C are (0 cm, 10 cm), (10 cm, 10 cm), and (10 cm, 0 cm), respectively.
Solve: The coordinates of the center of mass are

$$x_{cm} = \frac{m_A x_A + m_B x_B + m_C x_C}{m_A + m_B + m_C} = \frac{(200 \text{ g})(0 \text{ cm}) + (300 \text{ g})(10 \text{ cm}) + (100 \text{ g})(10 \text{ cm})}{(200 \text{ g} + 300 \text{ g} + 100 \text{ g})} = 6.7 \text{ cm}$$
$$y_{cm} = \frac{m_A y_A + m_B y_B + m_C y_C}{m_A + m_B + m_C} = \frac{(200 \text{ g})(10 \text{ cm}) + (300 \text{ g})(10 \text{ cm}) + (100 \text{ g})(0 \text{ cm})}{(200 \text{ g} + 300 \text{ g} + 100 \text{ g})} = 8.3 \text{ cm}$$

12.8. Model: The balls are particles located at the ball's respective centers.
Visualize:



Solve: The center of mass of the two balls measured from the left hand ball is

$$x_{cm} = \frac{(100\text{ g})(0\text{ cm}) + (200\text{ g})(30\text{ cm})}{100\text{ g} + 200\text{ g}} = 20\text{ cm}$$

The linear speed of the 100 g ball is

$$v_1 = r\omega = x_{cm}\omega = (0.20\text{ m})(120\text{ rev/min})\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{\text{min}}{60\text{ s}}\right) = 2.5\text{ m/s}$$

12.9. Model: The earth is a rigid, spherical rotating body.
Solve: The rotational kinetic energy of the earth is $K_{rot} = \frac{1}{2}I\omega^2$. The moment of inertia of a sphere about its diameter (see Table 12.2) is $I = \frac{2}{5}M_{earth}R^2$ and the angular velocity of the earth is

$$\omega = \frac{2\pi\text{ rad}}{24\times 3600\text{ s}} = 7.27\times 10^{-5}\text{ rad/s}$$

Thus, the rotational kinetic energy is

$$K_{rot} = \frac{1}{2}\left(\frac{2}{5}M_{earth}R^2\right)\omega^2$$
$$= \frac{1}{5}(5.98\times 10^{24}\text{ kg})(6.37\times 10^6\text{ m})^2(7.27\times 10^{-5}\text{ rad/s})^2 = 2.57\times 10^{29}\text{ J}$$

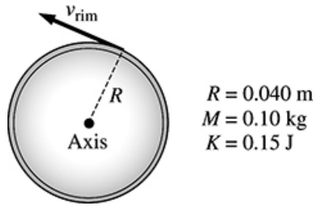
12.10. Model: The triangle is a rigid body rotating about an axis through the center.
Visualize: Please refer to Figure EX12.10. Each 200 g mass is a distance *r* away from the axis of rotation, where *r* is given by

$$\frac{0.20\text{ m}}{r} = \cos 30° \Rightarrow r = \frac{0.20\text{ m}}{\cos 30°} = 0.2309\text{ m}$$

Solve: The moment of inertia of the triangle is $I = 3\times mr^2 = 3(0.200\text{ kg})(0.2309\text{ m})^2 = 0.0320\text{ kg m}^2$. The frequency of rotation is given as 5.0 revolutions per s or 10π rad/s. The rotational kinetic energy is

$$K_{rot.} = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.0320\text{ kg m}^2)(10.0\pi\text{ rad/s})^2 = 15.8\text{ J}$$

12.11. Model: The disk is a rigid body rotating about an axis through its center.
Visualize:



Solve: The speed of the point on the rim is given by $v_{rim} = R\omega$. The angular velocity ω of the disk can be determined from its rotational kinetic energy which is $K = \frac{1}{2}I\omega^2 = 0.15$ J. The moment of inertia *I* of the disk about its center and perpendicular to the plane of the disk is given by

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(0.10\text{ kg})(0.040\text{ m})^2 = 8.0\times10^{-5}\text{ kg m}^2$$

$$\Rightarrow \omega^2 = \frac{2(0.15\text{ J})}{I} = \frac{0.30\text{ J}}{8.0\times10^{-5}\text{ kg m}^2} \Rightarrow \omega = 61.237\text{ rad/s}$$

Now, we can go back to the first equation to find v_{rim}. We get $v_{\text{rim}} = R\omega = (0.040\text{ m})(61.237\text{ rad/s}) = 2.4\text{ m/s}$.

12.12. Model: The baton is a thin rod rotating about a perpendicular axis through its center of mass.

Solve: The moment of inertia of a thin rod rotating about its center is $I = \frac{1}{12}ML^2$. For the baton,

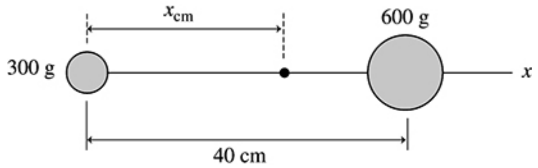
$$I = \frac{1}{12}(0.400\text{ kg})(0.96\text{ m})^2 = 0.031\text{ kg m}^2$$

The rotational kinetic energy of the baton is

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.031\text{ kg m}^2)\left((100\text{ rev/min})\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{\text{min}}{60\text{ s}}\right)\right)^2 = 1.68\text{ J}$$

12.13. Model: The structure is a rigid body rotating about its center of mass.

Visualize:



We placed the origin of the coordinate system on the 300 g ball.

Solve: First, we calculate the center of mass:

$$x_{\text{cm}} = \frac{(300\text{ g})(0\text{ cm}) + (600\text{ g})(40\text{ cm})}{300\text{ g} + 600\text{ g}} = 26.67\text{ cm}$$

Next, we will calculate the moment of inertia about the structure's center of mass:

$$I = (300\text{ g})(x_{\text{cm}})^2 + (600\text{ g})(40\text{ cm} - x_{\text{cm}})^2$$
$$= (0.300\text{ kg})(0.2667\text{ m})^2 + (0.600\text{ kg})(0.1333\text{ m})^2 = 0.032\text{ kg m}^2$$

Finally, we calculate the rotational kinetic energy:

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.032\text{ kg m}^2)\left(\frac{100\times2\pi}{60}\text{ rad/s}\right)^2 = 1.75\text{ J}$$

12.14. Model: The moment of inertia of any object depends on the axis of rotation. In the present case, the rotation axis passes through mass A and is perpendicular to the page.

Visualize: Please refer to Figure EX12.14.

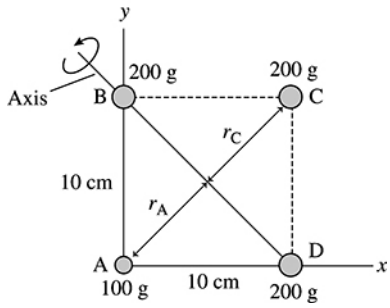
Solve: **(a)**
$$x_{\text{cm}} = \frac{\sum m_i x_i}{\sum m_i} = \frac{m_A x_A + m_B x_B + m_C x_C + m_D x_D}{m_A + m_B + m_C + m_D}$$
$$= \frac{(100\text{ g})(0\text{ m}) + (200\text{ g})(0\text{ m}) + (200\text{ g})(0.10\text{ m}) + (200\text{ g})(0.10\text{ m})}{100\text{ g} + 200\text{ g} + 200\text{ g} + 200\text{ g}} = 0.057\text{ m}$$
$$y_{\text{cm}} = \frac{m_A y_A + m_B y_B + m_C y_C + m_D y_D}{m_A + m_B + m_C + m_D}$$
$$= \frac{(100\text{ g})(0\text{ m}) + (200\text{ g})(0.10\text{ m}) + (200\text{ g})(0.10\text{ cm}) + (200\text{ g})(0\text{ m})}{700\text{ g}} = 0.057\text{ m}$$

(b) The distance from the axis to mass C is 14.14 cm. The moment of inertia through A and perpendicular to the page is

$$I_A = \sum_i m_i r_i^2 = m_A r_A^2 + m_B r_B^2 + m_C r_C^2 + m_D r_D^2$$
$$= (0.100\text{ kg})(0\text{ m})^2 + (0.200\text{ kg})(0.10\text{ m})^2 + (0.200\text{ kg})(0.1414\text{ m})^2 + (0.200\text{ kg})(0.10\text{ m})^2 = 0.0080\text{ kg m}^2$$

12.15. Model: The moment of inertia of any object depends on the axis of rotation.

Visualize:



Solve: **(a)** $$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{m_A x_A + m_B x_B + m_C x_C + m_D x_D}{m_A + m_B + m_C + m_D}$$

$$= \frac{(100 \text{ g})(0 \text{ m}) + (200 \text{ g})(0 \text{ m}) + (200 \text{ g})(0.10 \text{ m}) + (200 \text{ g})(0.10 \text{ m})}{100 \text{ g} + 200 \text{ g} + 200 \text{ g} + 200 \text{ g}} = 0.057 \text{ m}$$

$$y_{cm} = \frac{m_A y_A + m_B y_B + m_C y_C + m_D y_D}{m_A + m_B + m_C + m_D}$$

$$= \frac{(100 \text{ g})(0 \text{ m}) + (200 \text{ g})(0.10 \text{ m}) + (200 \text{ g})(0.10 \text{ cm}) + (200 \text{ g})(0 \text{ m})}{700 \text{ g}} = 0.057 \text{ m}$$

(b) The moment of inertia about a diagonal that passes through B and D is

$$I_{BD} = m_A r_A^2 + m_C r_C^2$$

where $r_A = r_C = (0.10 \text{ m})\cos 45° = 7.07 \text{ cm}$ and are the distances from the diagonal. Thus,

$$I_{BD} = (0.100 \text{ kg})r_A^2 + (0.200 \text{ kg})r_C^2 = 0.0015 \text{ kg m}^2$$

Assess: Note that the masses B and D, being on the axis of rotation, do not contribute to the moment of inertia.

12.16. Model: The three masses connected by massless rigid rods is a rigid body.
Visualize: Please refer to Figure EX12.16.

Solve: **(a)** $$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(0.100 \text{ kg})(0 \text{ m}) + (0.200 \text{ kg})(0.06 \text{ m}) + (0.100 \text{ kg})(0.12 \text{ m})}{0.100 \text{ kg} + 0.200 \text{ kg} + 0.100 \text{ kg}} = 0.060 \text{ m}$$

$$y_{cm} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(0.100 \text{ kg})(0 \text{ m}) + (0.200 \text{ kg})\left(\sqrt{(0.10 \text{ m})^2 - (0.06 \text{ m})^2}\right) + (0.100 \text{ kg})(0 \text{ m})}{0.100 \text{ kg} + 0.200 \text{ kg} + 0.100 \text{ kg}} = 0.040 \text{ m}$$

(b) The moment of inertia about an axis through A and perpendicular to the page is

$$I_A = \sum m_i r_i^2 = m_B (0.10 \text{ m})^2 + m_C (0.10 \text{ m})^2 = (0.100 \text{ kg})[(0.10 \text{ m})^2 + (0.10 \text{ m})^2] = 0.0020 \text{ kg m}^2$$

(c) The moment of inertia about an axis that passes through B and C is

$$I_{BC} = m_A \left(\sqrt{(0.10 \text{ m})^2 - (0.06 \text{ m})^2}\right)^2 = 0.00128 \text{ kg m}^2$$

Assess: Note that mass m_A does not contribute to I_A, and the masses m_B and m_C do not contribute to I_{BC}.

12.17. Model: The door is a slab of uniform density.
Solve: **(a)** The hinges are at the edge of the door, so from Table 12.2,

$$I = \frac{1}{3}(25 \text{ kg})(0.91 \text{ m})^2 = 6.9 \text{ kg m}^2$$

(b) The distance from the axis through the center of mass along the height of the door is
$d = \left(\frac{0.91 \text{ m}}{2} - 0.15 \text{ m}\right) = 0.305 \text{ m}$. Using the parallel–axis theorem,

$$I = I_{cm} + Md^2 = \frac{1}{12}(25 \text{ kg})(0.91 \text{ m})^2 + (25 \text{ kg})(0.305 \text{ cm})^2 = 4.1 \text{ kg m}^2$$

Assess: The moment of inertia is less for a parallel axis through a point closer to the center of mass.

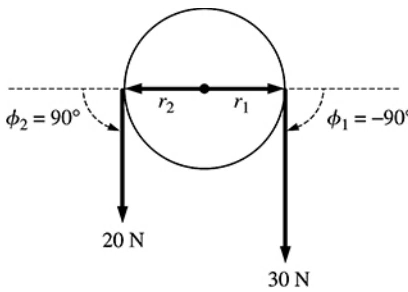
12.18. Model: The CD is a disk of uniform density.
Solve: **(a)** The center of the CD is its center of mass. Using Table 12.2,

$$I_{\text{cm}} = \frac{1}{2}MR^2 = \frac{1}{2}(0.021 \text{ kg})(0.060 \text{ m})^2 = 3.8\times10^{-5} \text{ kg m}^2$$

(b) Using the parallel–axis theorem with $d = 0.060$ m,

$$I = I_{\text{cm}} + Md^2 = 3.8\times10^{-5} \text{ kg m}^2 + (0.021 \text{ kg})(0.060 \text{ m})^2 = 1.14\times10^{-4} \text{ kg m}^2$$

12.19. Visualize:

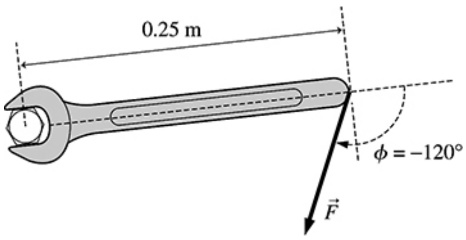


Solve: Torque by a force is defined as $\tau = Fr\sin\phi$ where ϕ is measured counterclockwise from the $\vec{r}$ vector to the $\vec{F}$ vector. The net torque on the pulley about the axle is the torque due to the 30 N force plus the torque due to the 20 N force:

$$(30 \text{ N})r_1 \sin\phi_1 + (20 \text{ N})r_2\sin\phi_2 = (30 \text{ N})(0.02 \text{ m}) \sin(-90°) + (20 \text{ N})(0.02 \text{ m}) \sin(90°)$$
$$= (-0.60 \text{ N m}) + (0.40 \text{ N m}) = -0.20 \text{ N m}$$

Assess: A negative torque causes a clockwise acceleration of the pulley.

12.21. Visualize:

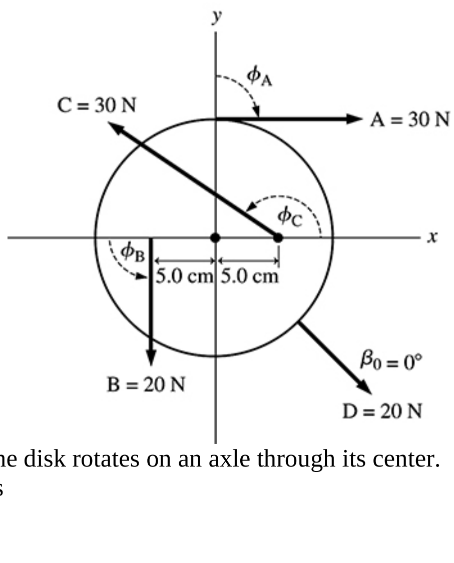


Solve: The net torque on the spark plug is

$$\tau = Fr\sin\phi = -38 \text{ N m} = F(0.25 \text{ m})\sin(-120°) \Rightarrow F = 176 \text{ N}$$

That is, you must pull with a force of 176 N to tighten the spark plug.
Assess: The force applied on the wrench leads to its clockwise motion. That is why we have used a negative sign for the net torque.

12.22. Model: The disk is a rotating rigid body.
Visualize:

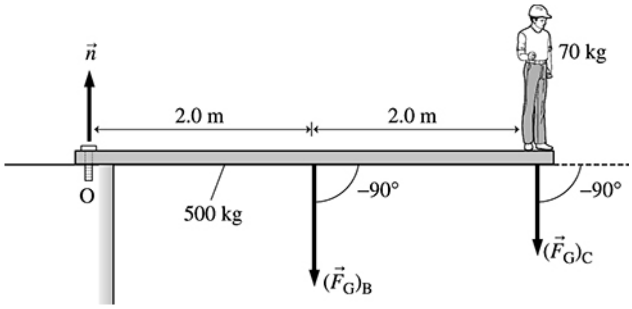


The radius of the disk is 10 cm and the disk rotates on an axle through its center.
Solve: The net torque on the axle is

$$\tau = F_A r_A \sin\phi_A + F_B r_B \sin\phi_B + F_C r_C \sin\phi_C + F_D r_D \sin\phi_D$$
$$= (30\text{ N})(0.10\text{ m})\sin(-90°) + (20\text{ N})(0.050\text{ m})\sin 90° + (30\text{ N})(0.050\text{ m})\sin 135° + (20\text{ N})(0.10\text{ m})\sin 0°$$
$$= -3\text{ N m} + 1\text{ N m} + 1.0607\text{ N m} = -0.94\text{ N m}$$

Assess: A negative torque means a clockwise rotation of the disk.

12.23. Model: The beam is a solid rigid body.
Visualize:



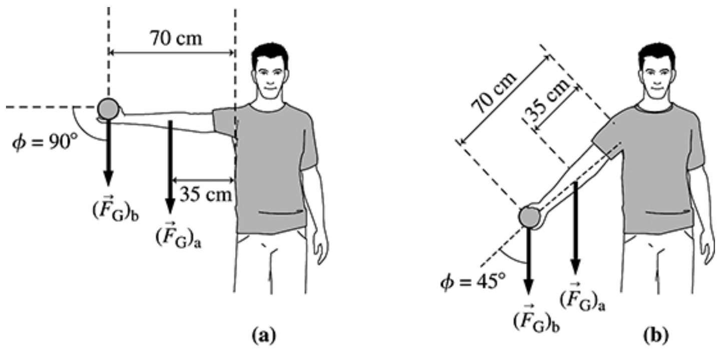
The steel beam experiences a torque due to the gravitational force on the construction worker $(\vec{F}_G)_C$ and the gravitational force on the beam $(\vec{F}_G)_B$. The normal force exerts no torque since the net torque is calculated about the point where the beam is bolted into place.

Solve: The net torque on the steel beam about point O is the sum of the torque due to $(\vec{F}_G)_C$ and the torque due to $(\vec{F}_G)_B$. The gravitational force on the beam acts at the center of mass.

$$\tau = ((F_G)_C)(4.0\text{ m})\sin(-90°) + ((F_G)_B)(2.0\text{ m})\sin(-90°)$$
$$= -(70\text{ kg})(9.80\text{ m/s}^2)(4.0\text{ m}) - (500\text{ kg})(9.80\text{ m/s}^2)(2.0\text{ m}) = -12.5\text{ kN m}$$

The negative torque means these forces would cause the beam to rotate clockwise. The magnitude of the torque is 12.5 kN m.

12.24. Model: Model the arm as a uniform rigid rod. Its mass acts at the center of mass.
Visualize:



Solve: **(a)** The torque is due both to the gravitational force on the ball and the gravitational force on the arm:

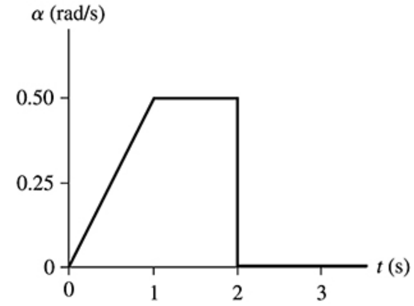
$$\tau = \tau_{ball} + \tau_{arm} = (m_b g) r_b \sin 90° + (m_a g) r_a \sin 90°$$
$$= (3.0\text{ kg})(9.8\text{ m/s}^2)(0.70\text{ m}) + (4.0\text{ kg})(9.8\text{ m/s}^2)(0.35\text{ m}) = 34\text{ N m}$$

(b) The torque is reduced because the moment arms are reduced. Both forces act at $\phi = 45°$ from the radial line, so

$$\tau = \tau_{ball} + \tau_{arm} = (m_b g) r_b \sin 45° + (m_a g) r_a \sin 45°$$
$$= (3.0\text{ kg})(9.8\text{ m/s}^2)(0.70\text{ m})(0.707) + (4.0\text{ kg})(9.8\text{ m/s}^2)(0.35\text{ m})(0.707) = 24\text{ N m}$$

12.25. Solve: $\tau = I\alpha$ is the rotational analog of Newton's second law $F = ma$. We have $\tau = (2.0\text{ kg m}^2)(4.0\text{ rad/s}^2) = 8.0\text{ kg m}^2/\text{s}^2 = 8.0\text{ N m}$.

12.26. Visualize: Since $\alpha = \tau/I$, a graph of the angular acceleration looks just like the torque graph with the numerical values divided by $I = 4.0 \text{ kg m}^2$.



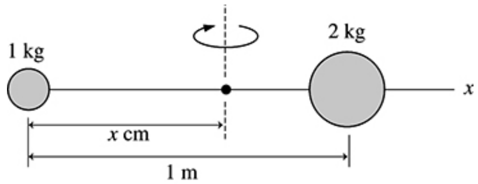
Solve: From the discussion about Figure 4.47

$$\omega_f = \omega_i + \text{area under the angular acceleration } \alpha \text{ curve between } t_i \text{ and } t_f$$

The area under the curve between $t = 0$ s and $t = 3$ s is 0.75 rad/s. With $\omega_1 = 0$ rad/s, we have

$$\omega_f = 0 \text{ rad/s} + 0.75 \text{ rad/s} = 0.75 \text{ rad/s}$$

12.27. Model: Two balls connected by a rigid, massless rod are a rigid body rotating about an axis through the center of mass. Assume that the size of the balls is small compared to 1 m.
Visualize:



We placed the origin of the coordinate system on the 1.0 kg ball.
Solve: The center of mass and the moment of inertia are

$$x_{cm} = \frac{(1.0 \text{ kg})(0 \text{ m}) + (2.0 \text{ kg})(1.0 \text{ m})}{(1.0 \text{ kg} + 2.0 \text{ kg})} = 0.667 \text{ m} \quad \text{and} \quad y_{cm} = 0 \text{ m}$$

$$I_{\text{about cm}} = \sum m_i r_i^2 = (1.0 \text{ kg})(0.667 \text{ m})^2 + (2.0 \text{ kg})(0.333 \text{ m})^2 = 0.667 \text{ kg m}^2$$

We have $\omega_f = 0$ rad/s, $t_f - t_i = 5.0$ s, and $\omega_i = -20 \text{ rpm} = -20(2\pi \text{ rad}/60 \text{ s}) = -\tfrac{2}{3}\pi \text{rad/s}$, so $\omega_f = \omega_i + \alpha(t_f - t_i)$ becomes

$$0 \text{ rad/s} = \left(-\frac{2\pi}{3} \text{rad/s}\right) + \alpha(5.0 \text{ s}) \Rightarrow \alpha = \frac{2\pi}{15} \text{rad/s}^2$$

Having found I and α, we can now find the torque τ that will bring the balls to a halt in 5.0 s:

$$\tau = I_{\text{about cm}}\alpha = \left(\frac{2}{3} \text{kg m}^2\right)\left(\frac{2\pi}{15} \text{ rad/s}^2\right) = \frac{4\pi}{45} \text{ N m} = 0.28 \text{ N m}$$

The magnitude of the torque is 0.28 N m, applied in the counterclockwise direction.

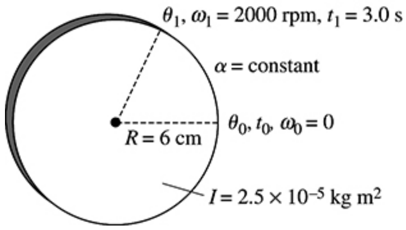
12.28. Model: A circular plastic disk rotating on an axle through its center is a rigid body. Assume axis is perpendicular to the disk.
Solve: To determine the torque (τ) needed to take the plastic disk from $\omega_i = 0$ rad/s to $\omega_f = 1800 \text{ rpm} = (1800)(2\pi)/60 \text{ rad/s} = 60\pi$ rad/s in $t_f - t_i = 4.0$ s, we need to determine the angular acceleration (α) and the disk's moment of inertia (I) about the axle in its center. The radius of the disk is $R = 10.0$ cm. We have

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(0.200 \text{ kg})(0.10 \text{ m})^2 = 1.0 \times 10^{-3} \text{ kg m}^2$$

$$\omega_f = \omega_i + \alpha(t_f - t_i) \Rightarrow \alpha = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{60\pi \text{ rad/s} - 0 \text{ rad/s}}{4.0 \text{ s}} = 15\pi \text{ rad/s}^2$$

Thus, $\tau = I\alpha = (1.0\times10^{-3}\text{ kg m}^2)(15\pi\text{ rad/s}^2) = 0.047\text{ N m}.$

12.29. Model: The compact disk is a rigid body rotating about its center.
Visualize:



Solve: **(a)** The rotational kinematic equation $\omega_1 = \omega_0 + \alpha(t_1 - t_0)$ gives

$$(2000\text{ rpm})\left(\frac{2\pi}{60}\right)\text{rad/s} = 0\text{ rad} + \alpha(3.0\text{ s} - 0\text{ s}) \Rightarrow \alpha = \frac{200\pi}{9}\text{ rad/s}^2$$

The torque needed to obtain this operating angular velocity is

$$\tau = I\alpha = (2.5\times10^{-5}\text{ kg m}^2)\left(\frac{200\pi}{9}\text{ rad/s}^2\right) = 1.75\times10^{-3}\text{ N m}$$

(b) From the rotational kinematic equation,

$$\theta_1 = \theta_0 + \omega_0(t_1 - t_0) + \frac{1}{2}\alpha(t_1 - t_0)^2 = 0\text{ rad} + 0\text{ rad} + \frac{1}{2}\left(\frac{200\pi}{9}\text{ rad/s}^2\right)(3.0\text{ s} - 0\text{ s})^2$$
$$= 100\pi\text{ rad} = \frac{100\pi}{2\pi}\text{ revolutions} = 50\text{ rev}$$

Assess: Fifty revolutions in 3 seconds is a reasonable value.

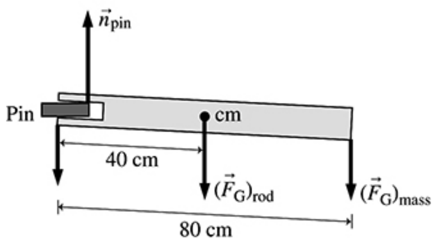
12.30. Model: The rocket attached to the end of a rigid rod is a rotating rigid body. Assume the rocket is small compared to 60 cm.
Visualize: Please refer to Figure EX12.30.
Solve: We can determine the rocket's angular acceleration from the relationship $\tau = I\alpha.$ The torque τ can be found from the thrust (F) using $\tau = Fr\sin\phi.$ The moment of inertia (I) can be calculated from equations given in Table 12.2. Specifically, $I = I_{\text{rod about one end}} + I_{\text{rocket}}$ becomes

$$\frac{1}{3}M_{\text{rod}}L^2 + ML^2 = \frac{1}{3}(0.100\text{ kg})(0.60\text{ m})^2 + (0.200\text{ kg})(0.60\text{ m})^2$$
$$= 0.012\text{ kg m}^2 + 0.072\text{ kg m}^2 = 0.0840\text{ kg m}^2$$

$$\Rightarrow \alpha = \frac{\tau}{I} = \frac{Fr\sin\phi}{I} = \frac{(4.0\text{ N})(0.60\text{ m})\sin(45°)}{0.0840\text{ kg m}^2} = 20\text{ rads/s}^2$$

Assess: The rocket will accelerate counterclockwise since α is positive.

12.31. Model: The rod is in rotational equilibrium, which means that $\tau_{\text{net}} = 0.$
Visualize:



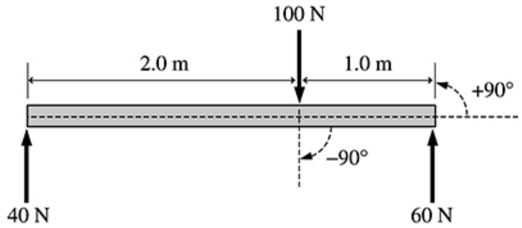
As the gravitational force on the rod and the hanging mass pull down (the rotation of the rod is exaggerated in the figure), the rod touches the pin at two points. The piece of the pin at the very end pushes down on the rod; the right end of the pin pushes up on the rod. To understand this, hold a pen or pencil between your thumb and forefinger, with your thumb on top (pushing down) and your forefinger underneath (pushing up).

Solve: Calculate the torque about the left end of the rod. The downward force exerted by the pin acts through this point, so it exerts no torque. To prevent rotation, the pin's normal force $\vec{n}_{\text{pin}}$ exerts a positive torque (ccw about the left end) to balance the negative torques (cw) of the gravitational force on the mass and rod. The gravitational force on the rod acts at the center of mass, so

$$\tau_{\text{net}} = 0 \text{ N m} = \tau_{\text{pin}} - (0.40 \text{ m})(2.0 \text{ kg})(9.8 \text{ m/s}^2) - (0.80 \text{ m})(0.50 \text{ kg})(9.8 \text{ m/s}^2)$$

$$\Rightarrow \tau_{\text{pin}} = 11.8 \text{ N m}$$

12.32. Model: The massless rod is a rigid body.
Visualize:

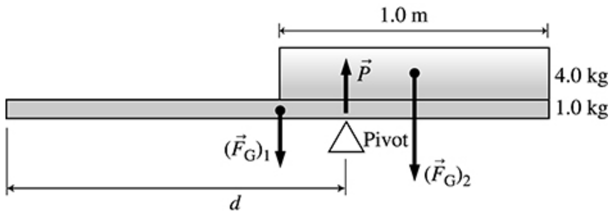


Solve: To be in equilibrium, the object must be in both translational equilibrium $(\vec{F}_{\text{net}} = 0 \text{ N})$ and rotational equilibrium $(\tau_{\text{net}} = 0 \text{ Nm})$. We have $(F_{\text{net}})_y = (40 \text{ N}) - (100 \text{ N}) + (60 \text{ N}) = 0 \text{ N}$, so the object is in translational equilibrium. Measuring τ_{net} about the left end,

$$\tau_{\text{net}} = (60 \text{ N})(3.0 \text{ m})\sin(+90°) + (100 \text{ N})(2.0 \text{ m})\sin(-90°) = -20 \text{ N m}$$

The object is not in equilibrium.

12.33. Model: The object balanced on the pivot is a rigid body.
Visualize:



Since the object is balanced on the pivot, it is in both translational equilibrium and rotational equilibrium.
Solve: There are three forces acting on the object: the gravitational force $(\vec{F}_G)_1$ acting through the center of mass of the long rod, the gravitational force $(\vec{F}_G)_2$ acting through the center of mass of the short rod, and the normal force $\vec{P}$ on the object applied by the pivot. The translational equilibrium equation $(F_{\text{net}})_y = 0 \text{ N}$ is

$$-(F_G)_1 - (F_G)_2 + P = 0 \text{ N} \Rightarrow P = (F_G)_1 + (F_G)_2 = (1.0 \text{ kg})(9.8 \text{ m/s}^2) + (4.0 \text{ kg})(9.8 \text{ m/s}^2) = 49 \text{ N}$$

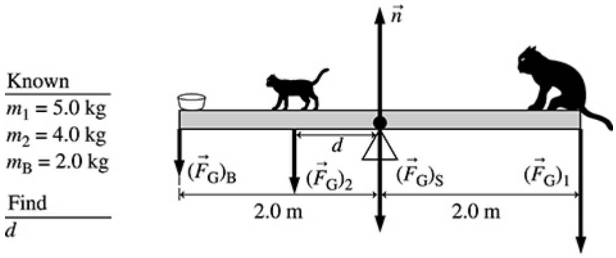
Measuring torques about the left end, the equation for rotational equilibrium $\tau_{\text{net}} = 0 \text{ Nm}$ is

$$Pd - w_1(1.0 \text{ m}) - w_2(1.5 \text{ m}) = 0 \text{ Nm}$$

$$\Rightarrow (49 \text{ N})d - (1.0 \text{ kg})(9.8 \text{ m/s}^2)(1.0 \text{ m}) - (4.0 \text{ kg})(9.8 \text{ m/s}^2)(1.5 \text{ m}) = 0 \text{ N} \Rightarrow d = 1.40 \text{ m}$$

Thus, the pivot is 1.40 m from the left end.

12.34. Model: The see-saw is a rigid body. The cats and bowl are particles.
Visualize:



Solve: The see-saw is in rotational equilibrium. Calculate the net torque about the pivot point.

$$\tau_{net} = 0 = (F_G)_1(2.0\text{ m}) - (F_G)_2(d) - (F_G)_B(2.0\text{ m})$$
$$m_2gd = m_1g(2.0\text{ m}) - m_Bg(2.0\text{ m})$$
$$d = \frac{(m_1 - m_B)(2.0\text{ m})}{m_2} = \frac{(5.0\text{ kg} - 2.0\text{ kg})(2.0\text{ m})}{4.0\text{ kg}} = 1.5\text{ m}$$

Assess: The smaller cat is close but not all the way to the end by the bowl, which makes sense since the combined mass of the smaller cat and bowl of tuna is greater than the mass of the larger cat.

12.35. Solve: **(a)** According to Equation 12.35, the speed of the center of mass of the tire is

$$v_{cm} = R\omega = 20\text{ m/s} \Rightarrow \omega = \frac{v_{cm}}{R} = \frac{20\text{ m/s}}{0.30\text{ m}} = 66.67\text{ rad/s} = (66.7)\left(\frac{60}{2\pi}\right)\text{ rpm} = 6.4\times10^2\text{ rpm}$$

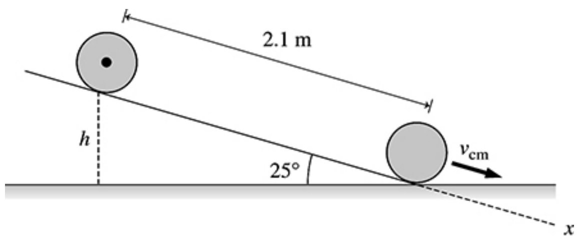
(b) The speed at the top edge of the tire relative to the ground is $v_{top} = 2v_{cm} = 2(20\text{ m/s}) = 40\text{ m/s}$.

(c) The speed at the bottom edge of the tire relative to ground is $v_{bottom} = 0$ m/s.

12.36. Model: The can is a rigid body rolling across the floor. Assume that the can has uniform mass distribution.
Solve: The rolling motion of the can is a translation of its center of mass plus a rotation about the center of mass. The moment of inertia of the can about the center of mass is $\frac{1}{2}MR^2$, where R is the radius of the can. Also $v_{cm} = R\omega$, where ω is the angular velocity of the can. The total kinetic energy of the can is

$$K = K_{cm} + K_{rot} = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_{cm}}{R}\right)^2$$
$$= \frac{3}{4}Mv_{cm}^2 = \frac{3}{4}(0.50\text{ kg})(1.0\text{ m/s})^2 = 0.38\text{ J}$$

12.37. Model: The sphere is a rigid body rolling down the incline without slipping.
Visualize:



The initial gravitational potential energy of the sphere is transformed into kinetic energy as it rolls down.
Solve: **(a)** If we choose the bottom of the incline as the zero of potential energy, the energy conservation equation will be $K_f = U_i$. The kinetic energy consists of both translational and rotational energy. This means

$$K_f = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}Mv_{cm}^2 = Mgh \Rightarrow \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2 + \frac{1}{2}M(R\omega)^2 = Mgh$$
$$\Rightarrow \frac{7}{10}MR^2\omega^2 = Mg(2.1\text{ m})\sin 25°$$
$$\Rightarrow \omega = \sqrt{\frac{\frac{10}{7}g(2.1\text{ m})(\sin 25°)}{R^2}} = \sqrt{\frac{\frac{10}{7}g(2.1\text{ m})(\sin 25°)}{(0.04\text{ m})^2}} = 88\text{ rad/s}$$

(b) From part (a)

$$K_{\text{total}} = \frac{1}{2}I_{\text{cm}}\omega^2 + \frac{1}{2}Mv_{\text{cm}}^2 = \frac{7}{10}MR^2\omega^2 \quad \text{and} \quad K_{\text{rot}} = \frac{1}{2}I_{\text{cm}}\omega^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2 = \frac{1}{5}MR^2\omega^2$$

$$\Rightarrow \frac{K_{\text{rot}}}{K_{\text{total}}} = \frac{\frac{1}{5}MR^2\omega^2}{\frac{7}{10}MR^2\omega^2} = \frac{1}{5}\times\frac{10}{7} = \frac{2}{7}$$

12.47. Visualize: Please refer to Figure EX12.47.

Solve: $\vec{L} = \vec{r}\times m\vec{v} = (3.0\hat{i} + 2.0\hat{j})\text{ m}\times(0.1\text{ kg})(4.0\hat{j})\text{ m/s}$

$$= 1.20(\hat{i}\times\hat{j})\text{ kg m}^2/\text{s} + 0.8(\hat{j}\times\hat{j})\text{ kg m}^2/\text{s} = 1.20\hat{k}\text{ kg m}^2/\text{s} + 0\text{ kg m}^2/\text{s}$$

$$= 1.20\hat{k}\text{ kg m}^2/\text{s or } (1.20\text{ kg m}^2/\text{s, out of page})$$

12.49. Model: The disk is a rotating rigid body.

Visualize: Please refer to Figure EX12.49.

Solve: From Table 12.2, the moment of inertial of the disk about its center is

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(2.0\text{ kg})(0.020\text{ m})^2 = 4.0\times10^{-4}\text{ kg m}^2$$

The angular velocity ω is $600\text{ rpm} = 600\times2\pi/60\text{ rad/s} = 20\pi\text{ rad/s}$. Thus, $L = I\omega = (4.0\times10^{-4}\text{ kg m}^2)(20\pi\text{ rad/s}) = 0.025\text{ kg m}^2/\text{s}$. If we wrap our right fingers in the direction of the disk's rotation, our thumb will point in the $-x$ direction. Consequently,

$$\vec{L} = -0.025\,\hat{i}\text{ kg m}^2/\text{s} = (0.025\text{ kg m}^2/\text{s, into page})$$

12.56. Model: The object is a rigid rotating body. Assume the masses m_1 and m_2 are small and the rod is thin.

Visualize: Please refer to P12.56.

Solve: The moment of inertia of the object is the sum of the moment of inertia of the rod, mass m_1, and mass m_2. Using Table 12.2 for the moment of inertia of the rod, we get

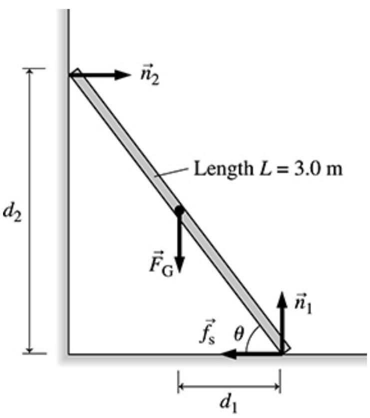
$$I_{\text{rod}} = I_{\text{rod about center}} + I_{m_1} + I_{m_2} = \frac{1}{12}ML^2 + m_1\left(\frac{L}{2}\right)^2 + m_2\left(\frac{L}{4}\right)^2$$

$$= \frac{1}{12}ML^2 + \frac{1}{4}m_1L^2 + \frac{1}{16}m_2L^2 = \frac{L^2}{4}\left(\frac{M}{3} + m_1 + \frac{m_2}{4}\right)$$

Assess: With $m_1 = m_2 = 0$ kg, $I_{\text{rod}}\,\frac{1}{12}ML^2$, as expected.

12.61. Model: The ladder is a rigid rod of length L. To not slip, it must be in both translational equilibrium $(\vec{F}_{\text{net}} = \vec{0}\text{ N})$ and rotational equilibrium $(\tau_{\text{net}} = 0\text{ N m})$. We also apply the model of static friction.

Visualize:



Since the wall is frictionless, the only force from the wall on the ladder is the normal force $\vec{n}_2$. On the other hand, the floor exerts both the normal force $\vec{n}_1$ and the static frictional force $\vec{f}_s$. The gravitational force $\vec{F}_G$ on the ladder acts through the center of mass of the ladder.

Solve: The x- and y-components of $\vec{F}_{\text{net}} = \vec{0}$ N are

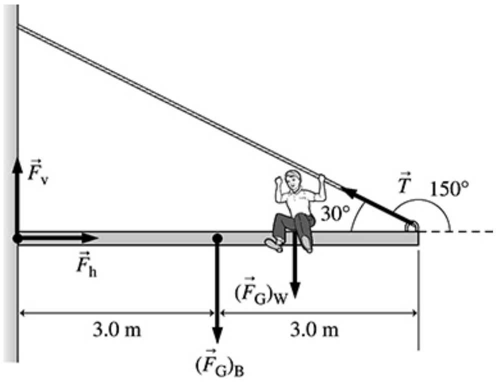
$$\sum F_x = n_2 - f_s = 0\text{ N} \Rightarrow f_s = n_2 \qquad \sum F_y = n_1 - F_G = 0\text{ N} \Rightarrow n_1 = F_G$$

The minimum angle occurs when the static friction is at its maximum value $f_{s\,max} = \mu_s n_1$. Thus we have $n_2 = f_s = \mu_s n_1 = \mu_s mg$. We choose the bottom corner of the ladder as a pivot point to obtain τ_{net}, because two forces pass through this point and have no torque about it. The net torque about the bottom corner is

$$\tau_{net} = d_1 mg - d_2 n_2 = (0.5L\cos\theta_{min})mg - (L\sin\theta_{min})\mu_s mg = 0 \text{ N m}$$

$$\Rightarrow 0.5\cos\theta_{min} = \mu_s \sin\theta_{min} \Rightarrow \tan\theta_{min} = \frac{0.5}{\mu_s} = \frac{0.5}{0.4} = 1.25 \Rightarrow \theta_{min} = 51°$$

12.63. Model: The structure is a rigid body.
Visualize:

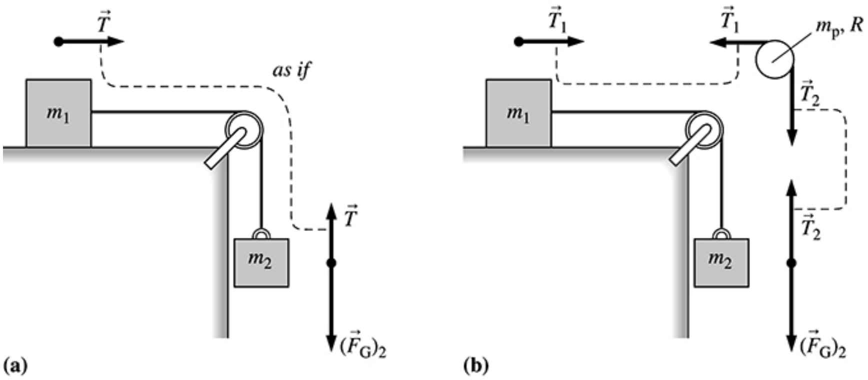


Solve: We pick the left end of the beam as our pivot point. We don't need to know the forces F_h and F_v because the pivot point passes through the line of application of F_h and F_v and therefore these forces do not exert a torque. For the beam to stay in equilibrium, the net torque about this point is zero. We can write

$$\tau_{\text{about left end}} = -(F_G)_B(3.0 \text{ m}) - (F_G)_W(4.0 \text{ m}) + (T\sin 150°)(6.0 \text{ m}) = 0 \text{ N m}$$

Using $(F_G)_B = (1450 \text{ kg})(9.8 \text{ m/s}^2)$ and $(F_G)_W = (80 \text{ kg})(9.8 \text{ m/s}^2)$, the torque equation can be solved to yield $T = 15{,}300$ N. The tension in the cable is slightly more than the cable rating. The worker should be worried.

12.71. Model: Assume the string does not slip on the pulley.
Visualize:



The free-body diagrams for the two blocks and the pulley are shown. The tension in the string exerts an upward force on the block m_2, but a downward force on the outer edge of the pulley. Similarly the string exerts a force on block m_1 to the right, but a leftward force on the outer edge of the pulley.

Solve: (a) Newton's second law for m_1 and m_2 is $T = m_1 a_1$ and $T - m_2 g = m_2 a_2$. Using the constraint $-a_2 = +a_1 = a$, we have $T = m_1 a$ and $-T + m_2 g = m_2 a$. Adding these equations, we get $m_2 g = (m_1 + m_2)a$, or

$$a = \frac{m_2 g}{m_1 + m_2} \Rightarrow T = m_1 a = \frac{m_1 m_2 g}{m_1 + m_2}$$

(b) When the pulley has mass *m*, the tensions $(T_1 \text{ and } T_2)$ in the upper and lower portions of the string are different. Newton's second law for m_1 and the pulley are:

$$T_1 = m_1 a \quad \text{and} \quad T_1 R - T_2 R = -I\alpha$$

We are using the minus sign with α because the pulley accelerates clockwise. Also, $a = R\alpha.$ Thus, $T_1 = m_1 a$ and

$$T_2 - T_1 = \frac{I}{R}\frac{a}{R} = \frac{aI}{R^2}$$

Adding these two equations gives

$$T_2 = a\left(m_1 + \frac{I}{R^2}\right)$$

Newton's second law for m_2 is $T_2 - m_2 g = m_2 a_2 = -m_2 a.$ Using the above expression for T_2,

$$a\left(m_1 + \frac{I}{R^2}\right) + m_2 a = m_2 g \Rightarrow a = \frac{m_2 g}{m_1 + m_2 + I/R^2}$$

Since $I = \frac{1}{2}m_{\text{p}}R^2$ for a disk about its center,

$$a = \frac{m_2 g}{m_1 + m_2 + \frac{1}{2}m_{\text{p}}}$$

With this value for *a* we can now find T_1 and T_2:

$$T_1 = m_1 a = \frac{m_1 m_2 g}{m_1 + m_2 + \frac{1}{2}m_{\text{p}}} \qquad T_2 = a(m_1 + I/R^2) = \frac{m_2 g}{\left(m_1 + m_2 + \frac{1}{2}m_{\text{p}}\right)}\left(m_1 + \frac{1}{2}m_{\text{p}}\right) = \frac{m_2\left(m_1 + \frac{1}{2}m_{\text{p}}\right)g}{m_1 + m_2 + \frac{1}{2}m_{\text{p}}}$$

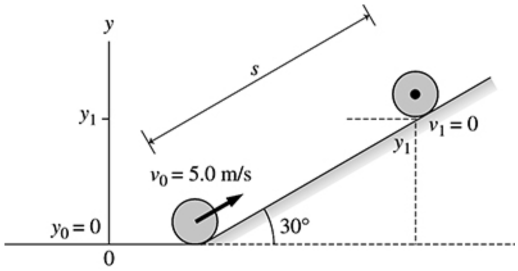
Assess: For $m = 0$ kg, the equations for *a*, T_1, and T_2 of part (b) simplify to

$$a = \frac{m_2 g}{m_1 + m_2} \quad \text{and} \quad T_1 = \frac{m_1 m_2 g}{m_1 + m_2} \quad \text{and} \quad T_2 = \frac{m_1 m_2 g}{m_1 + m_2}$$

These agree with the results of part (a).

12.74. Model: Assume that the hollow sphere is a rigid rolling body and that the sphere rolls up the incline without slipping. We also assume that the coefficient of rolling friction is zero.

Visualize:



The initial kinetic energy, which is a combination of rotational and translational energy, is transformed in gravitational potential energy. We chose the bottom of the incline as the zero of the gravitational potential energy.

Solve: The conservation of energy equation $K_{\text{f}} + U_{\text{gf}} = K_{\text{i}} + U_{\text{gi}}$ is

$$\frac{1}{2}M(v_1)^2_{\text{cm}} + \frac{1}{2}I_{\text{cm}}(\omega_1)^2 + Mgy_1 = \frac{1}{2}M(v_0)^2_{\text{cm}} + \frac{1}{2}I_{\text{cm}}(\omega_0)^2 + Mgy_0$$

$$0\text{ J} + 0\text{ J} + Mgy_1 = \frac{1}{2}M(v_0)^2_{\text{cm}} + \frac{1}{2}\left(\frac{2}{3}MR^2\right)(\omega_0)^2_{\text{cm}} + 0\text{ J} \Rightarrow Mgy_1 = \frac{1}{2}M(v_0)^2_{\text{cm}} + \frac{1}{3}MR^2\frac{(v_0)^2_{\text{cm}}}{R^2}$$

$$\Rightarrow gy_1 = \frac{5}{6}(v_0)^2_{\text{cm}} \Rightarrow y_1 = \frac{\frac{5}{6}(v_0)^2_{\text{cm}}}{g} = \frac{5}{6}\frac{(5.0\text{ m/s})^2}{9.8\text{ m/s}^2} = 2.126\text{ m}$$

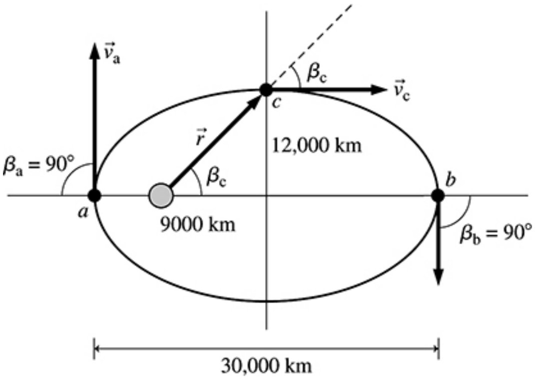
The distance traveled along the incline is

$$s = \frac{y_1}{\sin 30°} = \frac{2.126\text{ m}}{0.5} = 4.3\text{ m}$$

Assess: This is a reasonable stopping distance for an object rolling up an incline when its speed at the bottom of the incline is approximately 10 mph.

12.81. Model: The angular momentum of the satellite in the elliptical orbit is a constant.

Visualize:



Solve: **(a)** Because the gravitational force is always along the same direction as the direction of the moment arm vector, the torque $\vec{\tau} = \vec{r} \times \vec{F}_g$ is zero at all points on the orbit.
(b) The angular momentum of the satellite at any point on the elliptical trajectory is conserved. The velocity is perpendicular to $\vec{r}$ at points a and b, so $\beta = 90°$ and $L = mvr$. Thus

$$L_b = L_a \Rightarrow mv_b r_b = mv_a r_a \Rightarrow v_b = \left(\frac{r_a}{r_b}\right) v_a$$

$$r_a = \frac{30{,}000 \text{ km}}{2} - 9000 \text{ km} = 6000 \text{ km} \quad \text{and} \quad r_b = \frac{30{,}000 \text{ km}}{2} + 9000 \text{ km} = 24{,}000 \text{ km}$$

$$\Rightarrow v_b = \left(\frac{6000 \text{ km}}{24{,}000 \text{ km}}\right)(8000 \text{ m/s}) = 2000 \text{ m/s}$$

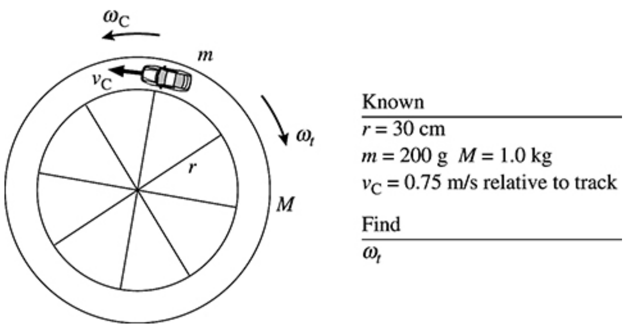
(c) Using the conservation of angular momentum $L_c = L_a$, we get

$$mv_c r_c \sin\beta_c = mv_a r_a \Rightarrow v_c = \left(\frac{r_a}{r_c}\right) v_a / \sin\beta_c \qquad r_c = \sqrt{(9000 \text{ km})^2 + (12{,}000 \text{ km})^2} = 1.5 \times 10^7 \text{ m}$$

From the figure, we see that $\sin\beta_c = 12{,}000/15{,}000 = 0.80$. Thus

$$v_c = \left(\frac{6000 \text{ km}}{15{,}000 \text{ km}}\right)\frac{(8000 \text{ m/s})}{0.80} = 4000 \text{ m/s}$$

12.89. Model: The toy car is a particle located at the rim of the track. The track is a cylindrical hoop rotating about its center, which is an axis of symmetry. No net torques are present on the track, so the angular momentum of the car and track is conserved.
Visualize:



Solve: The toy car's steady speed of 0.75 m/s relative to the track means that

$$v_c - v_t = 0.75 \text{ m/s} \Rightarrow v_c = v_t + 0.75 \text{ m/s},$$

where v_t is the velocity of a point on the track at the same radius as the car. Conservation of angular momentum implies that

$$L_i = L_f$$
$$0 = I_c\omega_c + I_t\omega_t = \left(mr^2\right)\omega_c + \left(Mr^2\right)\omega_t = m\omega_c + M\omega_t$$

The initial and final states refer to before and after the toy car was turned on. Table 12.2 was used for the track. Since $\omega_c = \frac{v_c}{r}$, $\omega_t = \frac{v_t}{r}$, we have

$$0 = mv_c + Mv_t$$
$$\Rightarrow m(v_t + 0.75\text{ m/s}) + Mv_t = 0$$
$$\Rightarrow v_t = -\frac{M}{m+M}(0.75\text{ m/s}) = -\frac{(0.200\text{ kg})}{(0.200\text{ kg} + 1.0\text{ kg})}(0.75\text{ m/s}) = -0.125\text{ m/s}$$

The minus sign indicates that the track is moving in the opposite direction of the car. The angular velocity of the track is

$$\omega_t = \frac{v_t}{r} = \frac{(0.125\text{ m/s})}{0.30\text{ m}} = 0.417\text{ rad/s clockwise.}$$

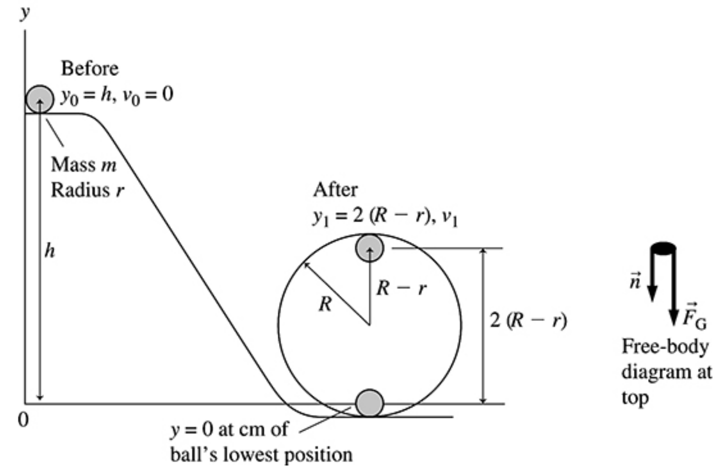
In rpm,

$$\omega_t = (0.417\text{ rad/s})\left(\frac{\text{rev}}{2\pi\text{rad}}\right)\left(\frac{60\text{ s}}{\text{min}}\right)$$
$$= 4.0\text{ rpm}$$

Assess: The speed of the track is less than that of the car because it is more massive.

12.91. Model: Assume that the marble does not slip as it rolls down the track and around a loop-the-loop. The mechanical energy of the marble is conserved.

Visualize:



Solve: The ball's center of mass moves in a circle of radius $R - r$. The free-body diagram on the marble at its highest position shows that Newton's second law for the marble is

$$mg + n = \frac{mv_1^2}{R - r}$$

The minimum height (h) that the track must have for the marble to make it around the loop-the-loop occurs when the normal force of the track on the marble tends to zero. Then the weight will provide the centripetal acceleration needed for the circular motion. For $n \to 0$ N,

$$mg = \frac{mv^2}{(R-r)} \Rightarrow v_1^2 = g(R-r)$$

Since rolling motion requires $v_1^2 = r^2\omega_1^2$, we have

$$\omega_1^2 r^2 = g(R-r) \Rightarrow \omega_1^2 = \frac{g(R-r)}{r^2}$$

The conservation of energy equation is

$$(K_f + U_{gf})_{\text{top of loop}} = (K_i + U_{gi})_{\text{initial}} \Rightarrow \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2 + mgy_1 = mgy_0 = mgh$$

Using the above expressions and $I = \frac{2}{5}mr^2$ the energy equation simplifies to

$$\frac{1}{2}mg(R-r) + \frac{1}{2}\left(\frac{2}{5}\right)mr^2\left(\frac{g(R-r)}{r^2}\right) + mg2(R-r) = mgh \Rightarrow h = 2.7(R-r)$$